TECHNICAL DISCUSSION

ON

DETERMINATION OF OSCILLATOR-CIRCUIT CONSTANTS IN SUPERHETERODYNE RECEIVERS

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DETERMINATION OF OSCILLATOR-CIRCUIT CONSTANTS IN SUPERHETERODYNE RECEIVERS

Superheterodyne receivers of today must be tuned with a single adjustment. The accomplishment of this with a receiver covering a certain frequency range requires that one or more tuned r-f circuits must be adjusted over that range while, at the same time, the oscillator circuit must be caused to "track" at a constant frequency difference from the r-f circuits. It is universal practice to use a "gang" of variable capacitors operated by a common shaft to tune both the signal and the oscillator circuit(s).

The most popular design of the present-day superheterodyne circuit includes for each of the signal circuits, a fixed inductor, an initially adjustable minimum capacitor, and a variable tuning capacitor. To obtain approximately the necessary frequency difference, the oscillator will include values of inductance and minimum capacitance different from those of the signal circuits, a fixed capacitor in series with the inductor, and a variable capacitor the same as those used with the signal circuits.

The present popularity of multirange receivers has increased the need for a simple method of designing these circuit elements. A simple method of design requiring no charts or other tools, except possibly a slide rule, is presented. This method is based on a mathematical analysis flexible enough to cover any frequency range and any intermediate frequency. It is sufficiently accurate to provide the required circuit values and to reduce greatly the necessity for experimental cuts and tries.

The circuits most frequently used to obtain alignment of oscillator and radio-frequency circuits are illustrated by Figures 1 and 2. Figure 1 represents the radio-frequency circuit consisting of inductance L and capacitance C. C is variable over a sufficient range to cover the desired range of frequencies. The minimum capacitance of the circuit, including that of the tubes and other elements, is considered as included in C. For the solution of the problem to be considered, it is necessary to know either the value of L or the value of C at some particular frequency. If the value of C at a frequency F_o is expressed as C_o, the corresponding value of L is given by the expression:

$$L = 25330/C_o F_o^2 \qquad (1)$$

in which L is expressed in microhenries, C_o in micro-microfarads, and F_o in megacycles. These are generally the most convenient units to use in the solution of the problem under consideration.

Figure 2 represents the oscillator circuit. In this circuit, the capacitance C is to be varied simultaneously with C of Figure 1 and is to have the same value as C of Figure 1 at all adjustments. The rate of variation of C with condenser setting is not important as long as the r-f tuning condenser and oscillator tuning condenser change at the same rate.

The capacitance C_3 represents the difference in minimum capacitance between the oscillator tuning capacitor and the radio-frequency tuning capacitor. Under certain conditions, the oscillator minimum may be the smaller (particularly, if C_4 is large); in this case, C_3 must be assigned a negative value. In most cases, C_3 will be positive. The capacitor C_2 is the series tracking capacitor, sometimes called the padding condenser. The capacitance C_4 is always present in a real circuit, since the distributed capacitance of the coil appears in this position. However, if C_4 is small compared to C_2, C_4 may be considered as a part of C_3. THIS MAY BE DONE IN PRACTICALLY ALL CASES IN WHICH THE INTERMEDIATE FREQUENCY IS LOWER THAN THE SIGNAL FREQUENCY.

When the required shunt capacitance C_4 is large and the series capacitance C_2 is small, C_4 must be taken into account. Also, in some cases, the adjustable shunt capacitor may be placed in the position of C_4. When C_4 must be considered, either C_3 or C_4 must be known approximately. This is not a serious restriction, since usually only one of these capacitances is adjustable. The other represents distributed capacitance of a coil, capacitance of wiring, input capacitance of a tube, or other similar factors and combinations; all of these may be estimated with sufficient accuracy. A small error in this estimate is compensated in practice by adjustment of the other shunt capacitor.

The desired relation between the resonant frequencies of the circuits of Figure 1 and Figure 2 is given by the relation:

$$f_1 = f + f_o \qquad (2)$$

Where, f_1 is the resonant frequency of the oscillator circuit
f is the resonant frequency of the radio-frequency circuit, and
f_o is the intermediate frequency.

When the circuit of Figure 2 is used, this relation cannot be satisfied at all frequencies. Instead, there are, in general, three frequencies at which equation (2) is exactly satisfied. The departure from this equation is in the regions between and just beyond these frequencies. The three values of the signal frequency (the "tracking frequencies") are designated as F_1, F_2, and F_3. These frequencies should be chosen in advance for each band under consideration. F_2 should be chosen near the center of this band; F_1 and F_3, for best results, should be placed near but not at the ends of the band.

On the following page is a summary of the formulas for calculating superheterodyne-oscillator constants. The Appendix contains the mathematical derivation of these formulas. For practical use in design problems, the summary sheet gives all necessary information to determine L_1, C_2, and C_3 or C_4 for the circuit shown in Figure 2. It is assumed that the following design information is available.

I. The intermediate frequency, f_o.
II. The tracking frequencies, F_1, F_2, F_3.

SUMMARY SHEET OF FORMULAS
FOR CALCULATION OF SUPERHETERODYNE OSCILLATOR CONSTANTS

Frequencies expressed in megacycles
Inductances " " microhenries
Capacitances " " micromicrofarads

Basic Considerations and Relations

f_o = Intermediate frequency

F_1, F_2, F_3 = Frequencies at which exact tracking is to be obtained.

$a = F_1 + F_2 + F_3$ (3)

$b^2 = F_1F_2 + F_1F_3 + F_2F_3$ (4)

$c^3 = F_1F_2F_3$ (5)

$d = a + 2f_o$ (6)

$l^2 = (b^2d - c^3)/2f_o$ (7)

$m^2 = l^2 + f_o^2 + ad - b^2$ (8)

$n^2 = (c^3d + f_o^2l^2)/m^2$ (9)

C_o = Tuning capacitance at frequency F_o

$L = 25330/C_oF_o^2$, or if L is known, then $C_oF_o^2 = 25330/L$ (1)

$A = C_oF_o^2(1/n^2 - 1/l^2)$ Required only for Case 3 (16)

$B = (C_oF_o^2/l^2) - C_3$ Required only for Case 4 (20)

R-F CIRCUIT

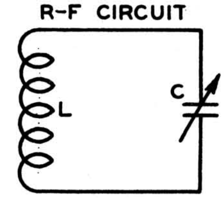

FIG.1

Case 1: When $C_4 = 0$, or $C_4 << C_2$ (the usual case).

$C_2 = C_oF_o^2(1/n^2 - 1/l^2)$ (10)

$C_3 = C_oF_o^2/l^2$ (11)

$L_1 = L(l^2/m^2)(C_2 + C_3)/C_2$ (12)

Case 2: When $C_3 = 0$.

$C_2 = C_oF_o^2/n^2$ (13)

$C_4 = C_oF_o^2/(l^2 - n^2)$ (14)

$L_1 = L(l^2/m^2)C_2/(C_2 + C_4)$ (15)

OSCILLATOR CIRCUIT

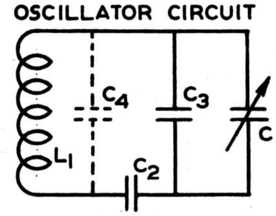

FIG.2

Case 3: When C_4 is known.

$C_2 = A\ (1/2 + \sqrt{1/4 + C_4/A}\ )$ (17)

$C_3 = (C_oF_o^2/l^2) - C_2C_4/(C_2 + C_4)$ (18)

$L_1 = L(l^2/m^2)(C_2 + C_3)/(C_2 + C_4)$ (19)

Case 4: When C_3 is known.

$C_2 = (C_oF_o^2/n^2) - C_3$ (21)

$C_4 = C_2B/(C_2 - B)$ (22)

$L_1 = L(l^2/m^2)(C_2 + C_3)/(C_2 + C_4)$ (23)

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Check Formulas

Equation for oscillator frequency:

$f_1 = m\sqrt{(f^2 + n^2)/(f^2 + l^2)}$ (24)

Equations for l^2, m^2, and n^2, in terms of oscillator constants:

$$l^2 = C_oF_o^2/(C_3 + \frac{C_2C_4}{C_2 + C_4}) \qquad (25)$$

$$m^2 = C_oF_o^2/(L_1/L)(C_4 + \frac{C_2C_3}{C_2 + C_3}) \qquad (26)$$

$$n^2 = C_oF_o^2/(C_2 + C_3) \qquad (27)$$

III. The inductance, L, or the capacitance, C_o, at frequency, F_o.
IV. The capacitance, C_4, or the capacitance, C_3.

Equations (24), (25), (26), and (27) may be used to check the results obtained. Also, these equations may be used to determine the oscillator frequency when circuit constants are known.

Two examples illustrating the use of these formulas follow:

Example I

Design a circuit covering the receiving frequency range from 150 kc to 420 kc and using an intermediate frequency of 465 kc.

Given: For frequency of 150 kc, C = 400 μμf.
Arbitrarily chosen: Tracking frequencies of 175 kc, 260 kc, and 380 kc.

Each of the cases will be worked out with given data as follows:

Case 1: Where $C_4 = 0$.

Case 2: Assume tube is connected across coil to give maximum voltage at grid of tube. C_4 will include tube input capacitance and distributed capacity of coil. Assume C_4 = 12 μμf.

Case 3: Assume that trimmer of capacitor has been adjusted to match minimum capacitance of r-f section. Consequently, $C_3 = 0$. The adjustment of minimum capacity is made by varying C_4.

Case 4: Same as Case 3, except that oscillator minimum trimmer has been removed from tuning capacitor. Minimum capacity of oscillator section is assumed to be 20 μμf less than the minimum of the r-f circuit. Consequently, C_3 = -20 μμf.

Frequencies are expressed in megacycles in order to give small numerical quantities.

Given: $f_o = 0.465$ $2f_o = 0.930$ $f_o^2 = 0.2162$
$F_1 = 0.175$ $F_2 = 0.260$ $F_3 = 0.380$
$F_o = 0.150$ $C_o = 400$ $C_oF_o^2 = 9.00$

Then: $a = 0.175 + 0.260 + 0.380 = 0.815$

$$b^2 = (0.175 \times 0.260) + (0.175 \times 0.380) + (0.260 \times 0.380)$$
$$= 0.0455 + 0.0665 + 0.0988 = 0.2108$$

$$c^3 = 0.175 \times 0.260 \times 0.380 = 0.01728$$

$$d = 0.815 + 0.930 = 1.745$$

$$l^2 = (0.2108 \times 1.745 - 0.0173)/0.930$$
$$= (0.3680 - 0.0175)/0.930 = 0.3507/0.930 = 0.377$$

$$m^2 = 0.815 \times 1.745 + 0.216 + 0.377 - 0.211$$
$$= 1.423 + 0.005 + 0.377 = 1.805$$

$$n^2 = (0.01728 \times 1.745 + 0.2162 \times 0.377)/1.805$$
$$= (0.0302 + 0.0815)/1.805 = 0.1117/1.805 = 0.0618$$

Case 1: $C_2 = 9.00(1/0.0618 - 1/0.377)$
$= 9.00(16.18 - 2.65) = 9.00 \times 13.53$ $= 121.8\ \mu\mu f$

$C_3 = 9.00/0.377$ $= 23.9\ \mu\mu f$

$L_1 = L\ (0.377/1.805)(145.7/121.8)$ $= 0.2496L$

Case 2: $C_4 = 12\ \mu\mu f$

$A = 9.0(1/0.0618 - 1/0.377)$ $= 121.8$
$C_2 = 121.8\ (\frac{1}{2} + \text{square root of quantity } \frac{1}{4} + 0.0985)$
$= 121.8\ (0.500 + \text{square root of } 0.3485)$
$= 121.8\ (0.500 + 0.590)$ $= 132.8\ \mu\mu f$

$C_3 = 9.00/0.377 - 12 \times 132.8/144.8 = 23.9 - 11.0$ $= 12.9\ \mu\mu f$

$L_1 = L\ (0.377/1.805)\ (145.7/144.8)$ $= 0.210L$

Case 3: $C_3 = 0$

$C_2 = 9.0/0.0618$ $= 145.7\ \mu\mu f$

$C_4 = 9.0/(0.377 - 0.062) = 9.0/0.315$ $= 28.5\ \mu\mu f$

$L_1 = L\ (0.377/1.805)\ (145.7/174.2)$ $= 0.2088L$

Case 4: $C_3 = -20$

$B = 9/0.377 + 20 = 23.88 + 20$ $= 43.88$

$C_2 = 9/0.0618 + 20 = 145.7 + 20$ $= 165.7\ \mu\mu f$

$C_4 = (165.7 + 43.88)/121.8$ $= 59.7\ \mu\mu f$

$L_1 = L\ (0.377/1.805)\ (1 + 5.7/225.4)$ $= 0.1348L$

The example chosen was selected to show wide divergency between values of C_2 and L_1/L for the various cases. When C_2 is larger, the differences between cases are much less pronounced.

The check formulas give for Case 4:

$l^2 = 9/(-20 + 43.8) \qquad = 0.377$

$m^2 = 9/0.1348(59.7 - 22.7) = 9/0.1348 \times 37.0 \qquad = 1.803$

$n^2 = 9/145.7 \qquad = 0.0618$

$f_1^2 = 1.805\ (f^2 + 0.0618)/(f^2 + 0.377)$

Check calculations tabulated for end frequencies, tracking frequencies, and two others:

f	0.150	0.175	0.220	0.260	0.320	0.380	0.420
f^2	0.0225	0.0306	0.0484	0.0576	0.1024	0.1444	0.1764
$f^2 + 0.0618$	0.0843	0.0924	0.1102	0.1294	0.1642	0.2062	0.2382
$f^2 + 0.377$	0.400	0.408	0.425	0.445	0.479	0.521	0.553
f_1^2	0.380	0.409	0.468	0.526	0.619	0.714	0.778
f_1	0.616	0.640	0.684	0.725	0.787	0.845	0.882
$f_1 - f$	0.466	0.465	0.464	0.465	0.467	0.465	0.462

The value $f_1 - f$ shows that tracking is very good except at the extreme high-frequency end.

Example II

Design a circuit covering the receiving range of 10 to 23 megacycles and using an intermediate frequency of 465 kc.

Since the signal frequency is considerably greater than the intermediate frequency, the equations of Case 1 ($C_4 = 0$) will be used.

Given: For frequency of 10 megacycles, C = 400 μμf.
Arbitrarily chosen: Tracking frequencies of 12, 16, and 20 megacycles.

Given:

$f_o = 465$	$2f_o = 0.93$	$f_o^2 = 0.216$
$F_1 = 0.12$	$F_2 = 18$	$F_3 = 20$
$F_o = 10$	$C_o = 400$	$C_oF_o^2 = 40000$

Then:

$a = 48$

$b^2 = 192 + 240 + 320 = 752$

$c^3 = 3840$

$d = 48.93$

$l^2 = 752 \times 48.93 - 3840 = 32960$

$m^2 = 234.8 + 0.2 + 32960 - 752 = 35560$

$$n^2 = (184000 + 712)/35560 = 5.19$$

$$C_2 = 40000\ (0.1927 - 0.00003) = 7708\ \mu\mu f$$

$$C_3 = 40000/32960 = 1.21\ \mu\mu f$$

$$L_1 = L\ (32960/35560)(7709/7708) = 0.927L$$

In this case, C_2 is so large compared with the tuning capacitor that C_2 could be omitted; a small correction of the inductance value would then be required. The resultant capacity of 400 μμf and 7700 μμf in series is 380 μμf; if the series capacity is omitted, inductance L_1 must be reduced to correspond to the increase in C. L_1/L is then equal to 0.927 x 380/400 = 0.88. An increase in the value of minimum capacity will complete the correction.

With the series capacity C_2 omitted, exact tracking occurs at only two points; however, tracking will be so close to the conditions of Example II that the difference is of little consequence.

APPENDIX

Derivation of Formulas

Referring to Figure 2, the total oscillator capacitance is

$$C_1 = C_4 + \frac{C_2(C_3 + C)}{C + C_3 + C_2} = \frac{C\,C_4 + C\,C_2 + C_2C_3 + C_2C_4 + C_3C_4}{C + C_2 + C_3} \tag{28}$$

$$L_1C_1 = L_1(C_2 + C_4)\cdot\frac{C + C_3 + C_2C_4/(C_2 + C_4)}{C + C_2 + C_3} \tag{29}$$

$$= L_1(C_2 + C_4)\cdot\frac{L\,C + L\,(C_3 + \frac{C_2C_4}{C_2 + C_4})}{L\,C + L\,(C_2 + C_3)} \tag{30}$$

$$= L_1(C_2 + C_4)\cdot\frac{L\,(C_3 + \frac{C_2C_4}{C_2 + C_4})}{L\,(C_2 + C_3)}\cdot\frac{L\,C/L\,(C_3 + \frac{C_2C_4}{C_2 + C_4}) + 1}{L\,C/L\,(C_2 + C_3) + 1} \tag{31}$$

$$= L_1\,\frac{C_2C_3 + C_2C_4 + C_3C_4}{C_2 + C_3}\cdot\frac{1/L\,(C_3 + \frac{C_2C_4}{C_2 + C_4}) + 1/L\,C}{1/L\,(C_2 + C_3) + 1/L\,C} \tag{32}$$

$$1/L_1C_1 = \frac{1}{L_1(C_4 + \frac{C_2C_3}{C_2 + C_3})}\cdot\frac{1/L\,C + 1/L\,(C_2 + C_3)}{1/L\,C + 1/L\,(C_3 + \frac{C_2C_4}{C_2 + C_4})} \tag{33}$$

All terms are now in the form $1/L\,C$, and are consequently equivalent to frequency terms of the form $(2\pi f)^2$.

Let $$(2\pi l)^2 = 1/L\,(C_3 + \frac{C_2C_4}{C_2 + C_4}) \tag{34}$$

$$(2\pi m)^2 = 1/L_1\,(C_4 + \frac{C_2C_3}{C_2 + C_3}) \tag{35}$$

$$(2\pi n)^2 = 1/L\,(C_2 + C_3) \tag{36}$$

Then, $$(2\pi f_1)^2 = (2\pi m)^2\,\frac{(2\pi f)^2 + (2\pi n)^2}{(2\pi f)^2 + (2\pi l)^2} \tag{37}$$

and $$f_1^2 = m^2\,(f^2 + n^2)/(f^2 + l^2) \tag{38}$$

This will be recognized as equation (24).

At the tracking frequencies,

$$f_1 = f + f_o,\ f_1^2 = f^2 + 2f\,f_o + f_o^2 \tag{39}$$

$$(f^2 + 2f\,f_o + f_o^2)(f^2 + l^2) = m^2(f^2 + n^2) \tag{40}$$

$$f^4 + 2f_o\,f^3 + (f_o^2 + l^2 - m^2)\,f^2 + 2f_o\,l^2\,f + f_o^2 l^2 - m^2 n^2 = 0 \tag{41}$$

This is a fourth degree equation. The real positive roots of this equation will be the tracking frequencies.

Let the four roots of this equation be F_1, F_2, F_3, F_4. Then, relations between roots and coefficients are:

$$F_1 + F_2 + F_3 + F_4 = -2f_o \tag{42}$$

$$F_1F_2 + F_1F_3 + F_2F_3 + F_4(F_1 + F_2 + F_3) = f_o^2 + l^2 - m^2 \tag{43}$$

$$F_1F_2F_3 + F_4(F_1F_2 + F_1F_3 + F_2F_3) = -2f_o\,l^2 \tag{44}$$

$$F_1F_2F_3F_4 = f_o^2\,l^2 - m^2\,n^2 \tag{45}$$

Since the sum of the roots is negative, at least one of the roots will be negative, and there can be no more than three frequencies at which perfect tracking may be obtained. Let these three frequencies be F_1, F_2 and F_3. Then,

$$F_4 = -(2f_o + F_1 + F_2 + F_3) \tag{46}$$

Let

$$a = F_1 + F_2 + F_3 \tag{47}$$

$$b^2 = F_1F_2 + F_1F_3 + F_2F_3 \tag{48}$$

$$c^3 = F_1F_2F_3 \tag{49}$$

$$d = F_1 + F_2 + F_3 + 2f_o \tag{50}$$

Then,

$$F_4 = -d \tag{51}$$

$$b^2 - ad = f_o^2 + l^2 - m^2 \tag{52}$$

$$c^3 - b^2 d = -2f_o\,l^2 \tag{53}$$

$$-c^3 d = f_o^2 l^2 - m^2 n^2 \tag{54}$$

Solving for l^2, m^2, n^2:

$$l^2 = (b^2 d - c^3)/2f_o \tag{55}$$

$$m^2 = ad + f_o^2 + l^2 - b^2 \tag{56}$$

$$n^2 = (f_o^2 l^2 + c^3\,d)/m^2 \tag{57}$$

These will be recognized as equations (7), (8) and (9).

When $f = F_o$, $C = C_o$ and

$$(2\pi F_o)^2 = 1/L\,C_o \tag{58}$$

Dividing equations (34), (35), (36) by (58),

$$l^2/F_o^2 = C_o/(C_3 + \frac{C_2C_4}{C_2 + C_4}) \quad (59)$$

$$m^2/F_o^2 = C_o/(L_1/L)(C_4 + \frac{C_2C_3}{C_2 + C_3}) \quad (60)$$

$$n^2/F_o^2 = C_o/(C_2 + C_3) \quad (61)$$

or

$$l^2 = C_oF_o^2/(C_3 + \frac{C_2C_4}{C_2 + C_4}) \quad (62)$$

$$m^2 = C_oF_o^2/(\frac{L_1}{L})(C_4 + \frac{C_2C_3}{C_2 + C_3}) \quad (63)$$

$$n^2 = C_oF_o^2/(C_2 + C_3) \quad (64)$$

These are the checking equations (25), (26) and (27).

$$l^2/m^2 = (\frac{L_1}{L})\ (C_4 + \frac{C_2C_3}{C_2 + C_3})/(C_3 + \frac{C_2C_4}{C_2 + C_4}) \quad (65)$$

$$= (\frac{L_1}{L})\ (C_2 + C_4)/(C_2 + C_3) \quad (66)$$

$$L_1/L = (l^2/m^2)(C_2 + C_3)/(C_2 + C_4) \quad (67)$$

This corresponds to equations (12), (15), (19), (23) .

For Case 1, $C_4 = 0$: (62) and (64) reduce to

$$l^2 = C_oF_o^2/C_3 \quad (68)$$

$$n^2 = C_oF_o^2/(C_2 + C_3) \quad (69)$$

or

$$C_3 = C_oF_o^2/l^2 \quad (70)$$

$$C_2 + C_3 = C_oF_o^2/n^2 \quad (71)$$

$$C_2 = C_oF_o^2\ (1/n^2 - 1/l^2) \quad (72)$$

(70) and (72) are the same as (11) and (10).

For Case 2, $C_3 = 0$:   (62) and (64) become

$$1^2 = C_0F_o^2/(\frac{C_2C_4}{C_2 + C_4}) \tag{73}$$

$$n^2 = C_0F_o^2/C_2 \tag{74}$$

and

$$C_2 = C_0F_o^2/n^2 \tag{75}$$

$$C_2C_4/(C_2 + C_4) = C_0F_o^2/1^2 \tag{76}$$

$$C_2C_4 = (C_0F_o^2/1^2)(C_2 + C_4) \tag{77}$$

$$C_4 = (C_0F_o^2/1^2)C_2/(C_2 - C_0F_o^2/1^2) \tag{78}$$

$$= (C_0F_o^2/1^2)(C_0F_o^2/n^2)/[C_0F_o^2/n^2 - C_0F_o^2/1^2] \tag{79}$$

$$= C_0F_o^2/(1^2 - n^2) \tag{80}$$

(75) and (80) are the same as (13) and (14) .

For Case 3, when C_4 is known: From (62) and (64),

$$1/1^2 = (C_3 + \frac{C_2C_4}{C_2 + C_4})/C_0F_o^2 \tag{81}$$

$$1/n^2 = (C_3 + C_2)/C_0F_o^2 \tag{82}$$

$$1/n^2 - 1/1^2 = (C_2 - \frac{C_2C_4}{C_2 + C_4})/C_0F_o^2 \tag{83}$$

Let

$$A = C_0F_o^2(1/n^2 - 1/1^2) = C_2 - \frac{C_2C_4}{C_2 + C_4} = C_2^2/(C_2 + C_4) \tag{84}$$

This is equivalent to equation (16).

Then ,

$$A(C_2 + C_4) = C_2^2 \tag{85}$$

$$C_2^2 - AC_2 - AC_4 = 0 \tag{86}$$

Solving for C_2,

$$C_2 = \frac{A}{2} \pm \sqrt{(\frac{A}{2})^2 + AC_4} \tag{87}$$

Since the second root is negative,

$$C_2 = A\ (\frac{1}{2} + \sqrt{\frac{1}{4} + C_4/A}\ ) \tag{88}$$

C_3 is best found from equation (81).

$$C_oF_o^2/1^2 = C_3 + \frac{C_2C_4}{C_2 + C_4} \qquad (89)$$

$$C_3 = C_oF_o^2/1^2 - \frac{C_2C_4}{C_2 + C_4} \qquad (90)$$

(88) and (90) are the same as (17) and (18).

For Case 4, when C_3 is known: Using (81) and (82),

$$C_2 = C_oF_o^2/n^2 - C_3 \qquad (91)$$

$$C_2C_4/(C_2 + C_4) = C_oF_o^2/1^2 - C_3 = B \qquad (92)$$

$$C_2C_4 = B(C_2 + C_4) \qquad (93)$$

$$C_4 = C_2B/(C_2 - B) \qquad (94)$$

(91), (92) and (94) are the same as (21), (20) and (22).